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Screening of Dirac flavor structure in the seesaw and neutrino mixing

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Abstract: We consider the mechanism of *screening* of the Dirac flavor structure in the context of the double seesaw mechanism. As a consequence of screening, the structure of the light neutrino mass matrix, m_ν, is determined essentially by the structure of the (Majorana) mass matrix, M_S, of new super-heavy (Planck scale) neutral fermions S. We calculate effects of the renormalization group running in order to investigate the stability of the screening mechanism with respect to radiative corrections. We find that screening is stable in the supersymmetric case, whereas in the standard model it is unstable for certain structures of M_S. The screening mechanism allows us to reconcile the (approximate) quark-lepton symmetry and the strong difference of the mixing patterns in the quark and lepton sectors. It opens new possibilities to explain a quasi-degenerate neutrino mass spectrum, special "neutrino" symmetries and quark-lepton complementarity. Screening can emerge from certain flavor symmetries or Grand Unification.

Keywords: Neutrino Physics, Beyond Standard Model, GUT.

Contents

1. Introduction

Quarks and leptons show an apparent correspondence which suggests their common origin [1]. The observation is based on the pattern of quantum numbers and the fact that both quarks and leptons come in three fermionic families (generations). This allows quarks and leptons to be embedded into unique multiplets associated with extended gauge symmetries [1−4]. In spite of lack of further experimental confirmations, the quark-lepton symmetry and unification [1−4] are still the most appealing concepts in physics beyond the Standard Model (SM).

Tiny neutrino masses emerge naturally from the seesaw mechanism [5, 6], which implies the existence of right handed neutrino fields ν_R and new high energy scale. The seesaw scale turns out to be numerically rather close to the scale of Grand Unification (GU), which emerges from the unification of gauge couplings. It seems as if both gauge and fermion unification point in the same direction of certain Grand Unified Theory, like SO(10) GUTs [3].

On the basis of quark-lepton unification (symmetry) one would expect similarities between the mass and mixing patterns of quarks and leptons. However, observations [7, 8] do not support this expectation. Indeed,

- Neutrino masses and mixings differ strongly from those in the quark sector. The hierarchy of neutrino masses, if exists, is weaker than the quark mass hierarchy. The 2-3 leptonic mixing is maximal or close to maximal, whereas the corresponding quark mixing is very small. The 1-2 leptonic mixing is large but not maximal and it seems smaller than the 2-3 leptonic mixing. In contrast, the 1-2 quark mixing - the largest quark mixing - is small. The only common feature is that the 1-3 mixings are small in both sectors.

- Data seem to indicate a particular "neutrino symmetry" [9] which does not show up in other sectors of the theory. This includes maximal or nearly maximal 2-3 mixing and relatively small 1-3 mixing. If the neutrino mass spectrum would turn out to be quasi-degenerate,[1] this could imply certain symmetry too. However, it is very difficult to extend the suggested symmetries not only onto the quarks but also to charged leptons and models (e.g. [11]) which realize such neutrino symmetries are quite involved.

- In the context of the seesaw mechanism, the observed values of neutrino masses require the masses of the right-handed (RH) neutrinos to be $M_N \lesssim 10^{14}$ GeV, i.e. two orders of magnitude below the Grand Unification scale, M_{GU}. Furthermore, the data implies a certain flavor structure and a hierarchy of the RH neutrino mass matrix which may indicate that the scale of RH neutrino masses emerges as a combination of the GU-scale and some other scale close to the Planck mass M_{Pl}:

$$M_N \sim \frac{M_{GU}^2}{M_{Pl}} \,. \tag{1.1}$$

- The experimental values of the quark and lepton mixing angles between the first and second generations appear to sum up to the maximal mixing angle:

$$\theta_{12} + \theta_C \ \approx \frac{\pi}{4}, \tag{1.2}$$

  with the leptonic mixing angle θ_{12} and the Cabibbo angle θ_C. The interpretation of such a quark-lepton complementarity relation [12 − 15] is rather controversial. If it is not a numerical accident, relation (1.2) implies some quark-lepton connection which is not present in ordinary GUTs. It may require some additional structure in the leptonic sector which produces maximal mixing and therefore has certain symmetry.

All these observations can be considered as hints against a simple unification of quarks and leptons.

[1] This would be necessary if the evidence for neutrino-less double beta decay [10] would be confirmed.

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There exist various attempts to reconcile the possible quark-lepton symmetry (unification) and the strong difference of patterns of masses and mixing in two sectors. The main problem is that the strong hierarchy of eigenvalues and small mixings of the quark mass matrices imply a certain hierarchy in the Dirac type Yukawa couplings which propagates due to the symmetry/unification to the lepton sector (see, however [16]). One possible solution is that dominant contribution to the neutrino masses, e.g., from the Yukawa coupling with a Higgs triplet, has no analogy in the quark sector. However, even in this case, it seems rather unnatural that some Yukawa interactions have a particular symmetry which is not realized for other Yukawa interactions. In the context of seesaw mechanism the observed features of neutrino masses and mixing can be related to a particular structure of the Majorana mass matrix of the RH neutrinos [17, 18].

In this paper in order to reconcile the observed pattern of the neutrino masses and mixings and quark-lepton symmetry, we elaborate on the double seesaw mechanism [19]. Specifically, we will discuss a version where the flavor structures of the Dirac type Yukawa couplings cancel completely [17, 20], in the light neutrino mass matrix. The properties of the neutrino mass matrix are entirely determined by physics (in particular, symmetries) at energies above the Grand unification scale. We will call this "screening of the Dirac flavor structure" or simply "screening" and we will show it may lead to a natural solution of the above mentioned problems.

The paper is organized as follows. In section 2 we describe the screening mechanism. We consider the renormalization group effects and the stability of the mechanism with respect to radiative corrections in section 3. Various applications of the mechanism are studied in section 4. In particular, we consider a possibility of the degenerate neutrino mass spectrum, existence of particular neutrino symmetries, and the quark-lepton complementarity. In section 5 we discuss how the screening mechanism can be realized in GUT and models with family symmetries. Conclusions are presented in section 6.

2. The flavor screening mechanism

In addition to the SM particles we introduce the three right-handed neutrinos, $\nu_{iR} \equiv N_i^c$, $(i = 1, 2, 3)$, three left-handed singlets S_i and Higgs boson singlet σ. The leptonic interactions are assumed to have the following features:

1. The SM leptonic doublets L_i have Yukawa couplings Y_ν with $N \equiv (\nu_R)^c$;

2. The Majorana mass terms of the right-handed neutrinos, N, are forbidden or strongly suppressed;

3. The right-handed neutrinos N_i have Yukawa couplings Y_N with singlets S_i and a Higgs boson σ;

4. The singlets S_i have Majorana mass terms, thus violating the lepton number.

Under these assumptions the following Yukawa interactions and mass terms appear in the lagrangian

$$-\mathscr{L} = L^T Y_\nu N \phi + S^T Y_N N \sigma + \frac{1}{2} S^T M_S S + L^T Y_e l^c \phi_d + \text{h.c.}\,, \tag{2.1}$$

(the flavor indices are omitted here). The last term in eq. 2.1 is the Yukawa coupling which generates masses for the charge leptons; in the Standard model: $\phi_d = \tilde{\phi} \equiv i\sigma_2 \phi^*$. After the Higgs bosons develop the non-zero VEVs, $\langle\phi\rangle$ and $\langle\sigma\rangle$, the neutrino mass matrix is generated in the basis (ν, N, S)

$$\mathcal{M} = \begin{pmatrix} 0 & Y_\nu \langle\phi\rangle & 0 \\ Y_\nu^T \langle\phi\rangle & 0 & Y_N^T \langle\sigma\rangle \\ 0 & Y_N \langle\sigma\rangle & M_S \end{pmatrix}. \tag{2.2}$$

For $\langle\phi\rangle \ll \langle\sigma\rangle \ll M_S$ it realizes the double (or cascade) seesaw mechanism [19]. (The case of singular matrices M_S will be considered separately in section 4.5). Block diagonalization of the matrix (2.2) leads to the mass matrix of light neutrinos

$$m_\nu^0 = \left[\frac{\langle\phi\rangle}{\langle\sigma\rangle}\right]^2 Y_\nu \left(Y_N\right)^{-1} M_S \left(Y_N^T\right)^{-1} Y_\nu^T\,. \tag{2.3}$$

This matrix has the remarkable feature that the Dirac type Yukawa coupling matrices cancel provided that Y_ν and Y_N are proportional, i.e., if

$$Y_\nu = c \cdot Y_N\,, \tag{2.4}$$

with constant c being typically of the order unity. (Its deviation from unity can be, e.g., due to the renormalization group effects.) As a result of the cancellation the mass matrix of light neutrinos becomes

$$m_\nu = c^2 \left[\frac{\langle\phi\rangle}{\langle\sigma\rangle}\right]^2 M_S\,, \tag{2.5}$$

where the neutrino mixings emerge entirely from the flavor structure in M_S [17]. The flavor structure of the Dirac type Yukawa couplings is completely eliminated, while it still determines all the features of the quark masses and mixings. We will call such a cancellation the *screening* of the Dirac flavor structure in the lepton mixing or briefly, screening. Screening is a consequence of the double seesaw (2.2) and the proportionality (2.4). These features can emerge from certain flavor symmetries or Grand Unification, as it will be discussed in section 5. The scale of neutrino masses is determined by the scales of the double seesaw. Taking

$$\langle\phi\rangle = v_{EW}\,, \qquad \langle\sigma\rangle \sim M_{GU} \sim 10^{16}\,\text{GeV}\,, \qquad M_S \sim M_{Pl} \tag{2.6}$$

one finds from (2.5)

$$m_\nu \sim \left[\frac{v_{EW}}{M_{GU}}\right]^2 M_{Pl}\,, \tag{2.7}$$

which lies in the phenomenologically required range, while the flavor structure of the neutrino mass matrix, and consequently the mass ratios and mixings are determined by the Planck scale physics.[2]

The Majorana mass matrix of the right handed neutrinos, M_N, generated after the first seesaw equals

$$M_N = -\langle\sigma\rangle^2 Y_N^T M_S^{-1} Y_N \,. \tag{2.8}$$

For the mass scales given in eq. (2.6) we obtain $M_N \sim M_{GU}^2/M_{Pl} \leq 10^{14}\,\text{GeV}$ which reproduces the relation (1.1) required by phenomenology.

An attractive scenario could be that some symmetry in the sector of singlets S exists. This symmetry leads to a particular and simple structure of M_S at high scales. The symmetry is broken at lower scales and incomplete screening, if exists, can be related to this breaking. In fact, phenomenology may require deviations from complete screening. Perturbations might, e.g., explain the observed mass splitting of the light neutrinos in the case of a degenerate spectrum of the singlets S_f. In this connection one can explore the following origins of perturbations (breaking) of the structure (2.2), (2.4):

- Nonzero elements in the 1-1, 1-3 and 2-2 blocks of matrix in eq. (2.2): These blocks have different gauge properties and the origin of these perturbations can be quite different.

- Mismatch between Y_ν and Y_N destroying cancellation.

- Perturbations in M_S. They may follow from the Planck scale physics. If the eigenvalues of M_S have some hierarchy then one needs to take into account the renormalization effects due to the Yukawa interactions of S, N and σ which influence the structure of matrix M_S.

A structure of the mass matrix for (ν, N, S) with non-zero 1-3 block (the direct νS-terms) has been considered in ref. [22]. Under certain conditions the matrix leads to the linear dependence (instead of quadratic or no dependence in our case) of the light neutrino mass matrix on the Dirac flavor structure. In a sense this realizes the half-screening effect.

Notice that one can introduce the Majorana mass matrix of the RH neutrinos in the form (2.8) immediately without invoking the double seesaw mechanism. This is done in [23] where form $M_N = -\langle\sigma\rangle^2 Y_N^T Y_N + \delta M_N$ with δM_N being the correction matrix has been *postulated*. In contrast, we propose a model for the structure (2.8) based on the double seesaw mechanism and additional flavor symmetry or/and Grand Unification.

[2]In the paper [21] the double seesaw matrix (2.2) has been considered with only one new singlet S (so that the Y_N is the 3 component column). However, such a matrix can not reproduce the required mass spectrum of light neutrinos for any structure of Y_N. The complete mass spectrum consists of three Majorana neutrinos with masses (in our notations) $\sim M_S$, $\sim \langle\sigma\rangle^2/M_S$, $\sim \langle\phi\rangle^2 M_S/\langle\sigma\rangle^2$, and two pseudo-Dirac neutrinos with masses at the electroweak scale: $\sim \langle\phi\rangle$. There is only one light neutrino. A possibility to get a correct spectrum in the model [21] is to introduce the direct non-negligible contribution to the RH neutrino masses (2-2 block) of the form $Y_\nu^T M' Y_\nu$, where M' is some 3×3 matrix to be tuned to fit the data.

3. Screening and renormalization group effect

In this section we will consider effects of the radiative corrections [24 – 27] on the screening mechanism. The proportionality relation (2.4) required for screening, holds presumably at the large scale of M_S or above that. However, the couplings Y_N and Y_ν have different gauge properties and their renormalization group running to the EW scale leads to different radiative corrections which may destroy the screening effect. On the other hand, small mismatch of the renormalized Yukawa matrices (partial screening) may explain some of the observed properties of the neutrino mass spectrum and mixings.

In what follows we will consider a minimal scenario — corrections due to the SM and MSSM interactions. Presence of other physics beyond the standard model can lead to additional renormalization effects.

3.1 Renormalization group effects

Here we will consider the renormalization group effects below a certain scale Λ which in turn is somehow below the masses of singlets M_S:

$$M_i \ll \Lambda \leq M_S \,, \tag{3.1}$$

here M_i are the masses of RH neutrinos (see figure 1). We consider M_S as the mass matrix of singlets at Λ, so that possible renormalization effects on M_S due to the couplings Y_N are included.

Let us stress that for the hierarchical Yukawa matrix Y_ν, and consequently Y_N, the mass spectrum of the RH neutrinos is (in general) strongly hierarchical: $M_N \sim Y_\nu^2$. Therefore effects of the RG running between different mass thresholds is crucial [28 – 30, 23, 31].

Let us introduce the effective operator $\overset{(n)}{O}_M$ which generates neutrino masses in the basis (ν, N)

$$\mathscr{L} = -(\nu^T, N^T)\, \overset{(n)}{O}_M \,(\nu^T, N^T)^T \,. \tag{3.2}$$

The superscript (n) designates the number of right-handed neutrinos which are not decoupled at a given energy scale, that is, neutrinos in the effective theory. This superscript will denote also a range of RG-running with a given number of RH neutrinos, and we use also the notation $\overset{(n-m)}{Z} \equiv \overset{(n)}{Z}\,\overset{(n+1)}{Z} \,\dots\, \overset{(m-1)}{Z}\,\overset{(m)}{Z}$.

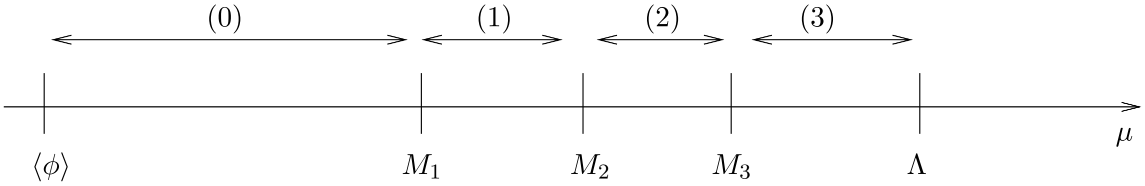

Figure 1: The thresholds due to masses of the right-handed neutrinos and the intervals of the RG running.

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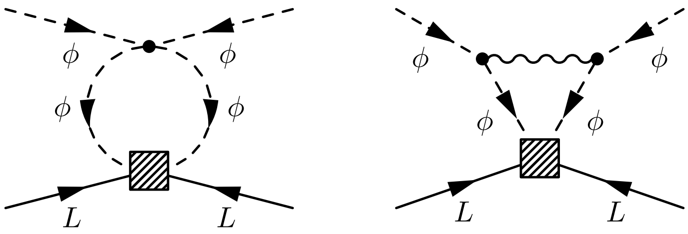

(a) Higgs self-coupling (b) Gauge interactions

Figure 2: The $d = 5$ operator vertex corrections. Shown are additional divergent diagrams in the effective theory.

Below the scale Λ the singlets S are integrated out, and the effective operator $\overset{(3)}{O}_M$ can be written as

$$\overset{(3)}{O}_M (\Lambda) = \begin{pmatrix} 0 & \overset{(3)}{Y}_\nu \phi \\ \overset{(3)}{Y}{}_\nu^T \phi & \overset{(3)}{M}_N \end{pmatrix} . \tag{3.3}$$

Here

$$\overset{(3)}{M}_N = -\langle\sigma\rangle^2 \, \overset{(3)}{Y}{}_N^T \, M_S^{-1} \, \overset{(3)}{Y}_N \tag{3.4}$$

is the mass matrix of the three RH neutrinos at the scale Λ, and $\overset{(3)}{Y}_\nu$ is the matrix of Yukawa couplings at Λ.

The effect of the RG evolution can be split in effects coming from the renormalization of the wave functions and the vertex corrections. It turns out, that the RG corrections can be factorized in the leading log (LL) approximation. So, in general, the renormalizations of Y_ν, M_N and $m_\nu \phi^2 / \langle\phi\rangle^2$ (operator of the light neutrino masses) are given by

$$Y_\nu \xrightarrow{\text{RG}} Z_{\text{ext}}^T Y_\nu Z_N \tag{3.5}$$

$$M_N \xrightarrow{\text{RG}} Z_N^T M_N Z_N \tag{3.6}$$

$$m_\nu \frac{\phi^2}{\langle\phi\rangle^2} \xrightarrow{\text{RG}} Z_{\text{ext}}^T m_\nu \frac{\phi^2}{\langle\phi\rangle^2} Z_{\text{ext}} Z_\kappa \, . \tag{3.7}$$

Here Z_{ext} combines the renormalization effect of the left-handed doublets L, the Higgs doublet ϕ, and the vertex correction to Y_ν; Z_N denotes the wave function renormalization effect of the RH neutrinos N. In order to simplify the presentation, we define the wave function renormalization so that the usual powers of 1/2 factors are absent. Eq. (3.7) describes renormalization of the effective dimension $d = 5$ operator which appears after decoupling (integration out) of the corresponding RH neutrino. Apart from renormalization of the wave functions and vertices which exist in the SM model this operator has additional vertex corrections given by the diagrams in figure 2. The RG effect due to these diagrams denoted by $\overset{(n)}{Z}_\kappa$ plays a crucial role in the discussion of the stability of screening. These $d = 5$ operator corrections are absent in the supersymmetric version of theory due to the non-renormalization theorem [32, 33].

We describe the RG effects in the effective theory, where the heavy RH neutrinos are decoupled successively[3] [28 – 30, 23, 31] as depicted in figure 1. In each step (interval between mass thresholds) we first calculate the RG correction to the matrices. We diagonalize the resulting matrices at the lower end of the interval, i.e., at $\mu = M_i$ and then decouple N_i, $(i = 3, 2, 1)$. We will denote the renormalization factors in the leading log approximation by $\overset{(n)}{Z} = 1 + \overset{(n)}{\delta Z}$. This notation is also used for parameters of the effective theory. The renormalization factors in the extended (by RH neutrinos) SM and the MSSM are given in the appendix A.

Let us describe the main steps of the renormalization procedure, which is especially simple in the basis where matrix Y_ν is diagonal. (This basis, however, does not coincide with the flavor basis.)

1. The RG evolution between Λ and M_3 yields the operator $\overset{(3)}{O}_M$ at M_3

$$\overset{(3)}{O}_M (M_3) = \begin{pmatrix} 0 & \overset{(3)}{Z}{}^T_{\rm ext} \overset{(3)}{Y}_\nu \, \phi \, \overset{(3)}{Z}_N \\ \ldots & \overset{(3)}{Z}{}^T_N \overset{(3)}{M}_N \overset{(3)}{Z}_N \end{pmatrix}. \tag{3.8}$$

Performing a rotation of the RH neutrinos $N = \overset{(3)}{U}_N N'$ we reduce the renormalized RH neutrino mass matrix to the form

$$\overset{(3)}{U}{}^T_N \overset{(3)}{Z}{}^T_N \overset{(3)}{M}_N \overset{(3)}{Z}_N \overset{(3)}{U}_N = \begin{pmatrix} \overset{(2)}{M}_N & 0 \\ 0 & M_3 \end{pmatrix}, \tag{3.9}$$

where $\overset{(2)}{M}_N$ is 2×2 (in general off-diagonal) mass matrix. Let us split the 3×3 Dirac type Yukawa coupling matrix in (3.8) after this rotation into two parts as

$$\overset{(3)}{Z}{}^T_{\rm ext} \overset{(3)}{Y}_\nu \overset{(3)}{Z}_N \overset{(3)}{U}_N \equiv \left(\overset{(2)}{Y}_\nu, \, y_3 \right), \tag{3.10}$$

where y_3 is the 3 component column of the Yukawa couplings of ν and N_3, and $\overset{(2)}{Y}_\nu$ is the rest 3×2 sub-matrix. Then in the rotated basis the operator (3.8) can be written as

$$\overset{(3)}{O}_M (M_3) = \begin{pmatrix} 0 & \overset{(2)}{Y}_\nu \, \phi & y_3 \phi \\ \ldots & \overset{(2)}{M}_N & 0 \\ & 0 & M_3 \end{pmatrix}. \tag{3.11}$$

[3]The running between mass thresholds of RH neutrinos has been treated analytically in the approximation of strongly hierarchical and diagonal Yukawa matrix in ref. [23]. Here we present a general consideration required for our approach.

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Below the scale M_3 the neutrino N_3 is integrated out and from (3.11) we obtain

$$\overset{(2)}{O}_M (M_3) = \begin{pmatrix} -y_3 M_3^{-1} y_3^T \phi^2 & \overset{(2)}{Y}_\nu \, \phi \\ \dots & \overset{(2)}{M}_N \end{pmatrix} . \tag{3.12}$$

Notice that the $d = 5$ operator is formed in 1-1 block due to decoupling of N_3.

2. Let us consider the RG running in the interval $M_2 - M_3$. Similarly to the first step we can write the operator O_M at the scale M_2 (threshold of N_2) as

$$\overset{(2)}{O}_M (M_2) = \begin{pmatrix} -\overset{(2)}{Z}{}^T_{\text{ext}} \overset{(2)}{Z}_\kappa \, y_3 M_3^{-1} y_3^T \phi^2 \, \overset{(2)}{Z}_{\text{ext}} & \overset{(2)}{Z}{}^T_{\text{ext}} \overset{(2)}{Y}_\nu \, \phi \, \overset{(2)}{Z}_N \\ \dots & \overset{(2)}{Z}{}^T_N \overset{(2)}{M} \overset{(2)}{Z}_N \end{pmatrix} . \tag{3.13}$$

Here we have included the corrections $\overset{(2)}{Z}_\kappa$ to the $d = 5$ operator.

By applying the rotation $N' = \overset{(2)}{U}_N \, N''$ the renormalized mass matrix of the RH neutrinos is diagonalized:

$$\overset{(2)}{U}{}^T_N \overset{(2)}{Z}{}^T_N \overset{(2)}{M} \overset{(2)}{Z}_N \overset{(2)}{U}_N \equiv \begin{pmatrix} \overset{(1)}{M}_N & 0 \\ 0 & M_2 \end{pmatrix} . \tag{3.14}$$

The renormalized Yukawa matrix is then split as

$$\overset{(2)}{Z}{}^T_{\text{ext}} \overset{(2)}{Y}_\nu \overset{(2)}{Z}_N \overset{(2)}{U}_N \equiv \left(\overset{(1)}{Y}_\nu , \; y_2 \right) , \tag{3.15}$$

where $\overset{(1)}{Y}_\nu$ and y_2 are two component columns. Decoupling the second neutrino N_2 we obtain

$$\overset{(1)}{O}_M (M_2) = \begin{pmatrix} -\overset{(2)}{Z}{}^T_{\text{ext}} \overset{(2)}{Z}_\kappa \, y_3 M_3^{-1} y_3^T \phi^2 \, \overset{(2)}{Z}_{\text{ext}} - y_2 M_2^{-1} y_2^T \phi^2 & \overset{(1)}{Y}_\nu \, \phi \\ \dots & \overset{(1)}{M}_N \end{pmatrix} . \tag{3.16}$$

3. Running the matrix down to the lowest see-saw scale M_1 and integration out N_1 yields

$$\begin{aligned} \overset{(0)}{O}_M (M_1) = & - \overset{(1-2)}{Z}{}^T_{\text{ext}} \overset{(1-2)}{Z}_\kappa \; y_3 M_3^{-1} y_3^T \phi^2 \, \overset{(1-2)}{Z}_{\text{ext}} \\ & - \overset{(1)}{Z}{}^T_{\text{ext}} \left[\overset{(1)}{Z}_\kappa \, y_2 M_2^{-1} y_2^T + \overset{(1)}{Y}_\nu \overset{(1)}{M}{}_N^{-1} \overset{(1)}{Y}{}_\nu^T \right] \phi^2 \, \overset{(1)}{Z}_{\text{ext}} \, . \end{aligned} \tag{3.17}$$

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4. Finally, evolving $\overset{(0)}{O}_M\ (M_1)$ down to the EW scale, we obtain (after ϕ develops a VEV) the mass matrix of light neutrinos

$$m_\nu = -\langle\phi\rangle^2\ \overset{(0-3)}{Z}{}^T_{\rm ext}\overset{(3)}{Y}_\nu \overset{(3)}{Z}_N \overset{(3)}{U}_N \begin{pmatrix} K_{12} & 0 \\ 0 & \frac{\overset{(0-2)}{Z_\kappa}}{M_3} \end{pmatrix} \overset{(3)}{U}{}^T_N \overset{(3)}{Z}{}^T_N \overset{(3)}{Y}{}^T_\nu \overset{(0-3)}{Z}_{\rm ext}\,, \tag{3.18}$$

with

$$K_{12} \equiv \overset{(2)}{Z}_N \overset{(2)}{U}_N \begin{pmatrix} \frac{\overset{(0)}{Z_\kappa}}{\overset{(1)}{M_N}} & 0 \\ 0 & \frac{\overset{(0-1)}{Z_\kappa}}{M_2} \end{pmatrix} \overset{(2)}{U}_N{}^T\ \overset{(2)}{Z}{}^T_N\ . \tag{3.19}$$

This expression can be presented in a simpler and transparent way. Using definitions of matrices $\overset{(2)}{U}_N$ and $\overset{(3)}{U}_N$ in eqs. (3.14), (3.9) we can rewrite m_ν as

$$m_\nu = -\langle\phi\rangle^2\, Z^T_{\rm ext} \left[\overset{(3)}{Y}_\nu\ X_N\ \overset{(3)}{M}{}^{-1}_N \overset{(3)}{Y}{}^T_\nu\right] Z_{\rm ext}\,, \tag{3.20}$$

where

$$X_N \equiv \overset{(3)}{Z}_N \overset{(3)}{U}_N \overset{(2)}{Z'}_N \overset{(2)}{U'}_N\ Z_\kappa\ \overset{(2)}{U'}{}^\dagger_N \overset{(2)}{Z'}{}^{-1}_N \overset{(3)}{U}{}^\dagger_N \overset{(3)}{Z}{}^{-1}_N \tag{3.21}$$

describes the RG effects due to the running between the thresholds. Here

$$\overset{(2)}{Z'}_N \equiv \begin{pmatrix} \overset{(2)}{Z}_N & 0 \\ 0 & 1 \end{pmatrix}, \qquad \overset{(2)}{U'}_N \equiv \begin{pmatrix} \overset{(2)}{U}_N & 0 \\ 0 & 1 \end{pmatrix}, \tag{3.22}$$

and

$$Z_\kappa \equiv \mathrm{diag}\left(\overset{(0)}{Z}_\kappa,\ \overset{(0-1)}{Z}_\kappa\ ,\ \overset{(0-2)}{Z}_\kappa\right) \tag{3.23}$$

is the matrix of effective $d = 5$ operator corrections (figure 2). The expressions for $Z_{\rm ext}$ are given in appendices A.1 and A.2. Formulas (3.18) and (3.20) for m_ν are our main results which we will analyze in the following sections.

3.2 Stability of screening

Inserting the expression for the matrix M_N, eq. (3.4), into eq. (3.20) we obtain

$$m_\nu = \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} Z^T_{\rm ext} \left[\overset{(3)}{Y}_\nu\ X_N\ \overset{(3)}{Y}{}^{-1}_N\ M_S\ \overset{(3)}{Y}{}^{T-1}_N \overset{(3)}{Y}{}^T_\nu\right] Z_{\rm ext}\,. \tag{3.24}$$

If the equality (2.4) is satisfied, the neutrino mass matrix becomes

$$m_\nu = \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} Z^T_{\rm ext} \left[\overset{(3)}{Y}_\nu\ X_N\ \overset{(3)}{Y}{}^{-1}_\nu\ M_S\right] Z_{\rm ext}\,. \tag{3.25}$$

Therefore screening is reproduced and the dependence of m_ν on the Yukawa (Dirac) couplings disappears if $X_N = I$. The expression (3.25) coincides with that in (2.5) up to external renormalization. In turn, according to eq. (3.21) the equality $X_N = I$ holds provided that $Z_\kappa = I$, that is, when the $d = 5$ operator corrections are absent. This is automatically satisfied in the supersymmetric theory, but the corrections are present in the SM and its non-supersymmetric extensions.

Note that the $d = 5$ operator corrections are due to the gauge interactions and self interactions of the Higgs boson, which are by themselves flavor blind. However, they influence the flavor structure of the light neutrino mass matrix due to difference of masses of the RH neutrinos and therefore different threshold effects.

3.3 Beyond leading log approximation

Beyond the leading log (LL) approximation the RG contributions still factorize except for the $d = 5$ operator corrections. Thus, our analysis does hold beyond LL in the MSSM, but it does not apply in the SM due to diagrams like in figure 3. However, the 2 loop contributions to the external renormalization are smaller than the 1 loop corrections by a factor of $y^2/16\pi^2 \leq 0.01$ and the 1 loop threshold corrections are not enhanced by large logarithms.

For certain structures of M_S (see section 4) the additional 2 loop diagrams lead to corrections to the renormalization of the effective neutrino mass operator which could be comparable to the 1 loop corrections. However, assuming the same hierarchy in the neutrino Yukawa couplings as in the up-type quark Yukawa couplings, these contribution are further suppressed since the heaviest right-handed neutrino is already integrated out. Therefore all 2 loop contributions in the effective theory are suppressed by $y_2^2/16\pi^2 \leq 10^{-6}$ compared to the 1 loop contributions. Altogether higher loop contributions are less than 10% of the 1 loop corrections and will be neglected in the following.

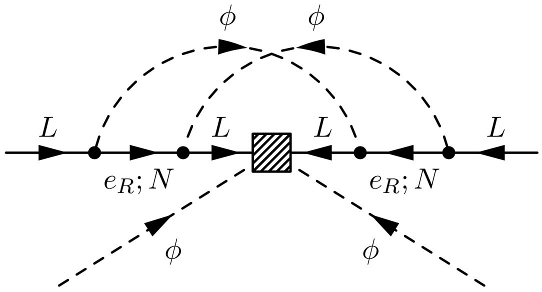

Figure 3: Examples of the two loop diagram which destroy factorization of the vertex corrections to the effective neutrino mass matrix.

3.4 Effects of vertex corrections

In order to study the $d = 5$ operator corrections in the (non-supersymmetric) standard model in more details we introduce the matrix V_N which diagonalizes the RH neutrino mass matrix at Λ:

$$V_N^T \overset{(3)}{M}_N V_N = \langle\sigma\rangle^2 V_N^T \overset{(3)}{Y}{}_N^T M_S^{-1} \overset{(3)}{Y}_N V_N = D_N \equiv \mathrm{diag}(M_1, M_2, M_3). \tag{3.26}$$

In the lowest order approximation: $\overset{(2)}{Z}_N = \overset{(3)}{Z}_N = 1$, and according to (3.9) and (3.14) we obtain

$$V_N = \overset{(3)}{U}_N \overset{(2)}{U'}_N \ . \tag{3.27}$$

Therefore the matrix X_N (3.21) can be rewritten in the form

$$X_N = V_N Z_\kappa V_N^\dagger = I + V_N \delta Z_\kappa V_N^\dagger \,, \tag{3.28}$$

where

$$\delta Z_\kappa \equiv Z_\kappa - I \,.$$

Plugging this expression for X_N in (3.24) we find

$$m_\nu \equiv \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} Z_{\rm ext}^T \left[I + \overset{(3)}{Y_\nu} V_N \delta Z_\kappa V_N^\dagger \overset{(3)}{Y_\nu}{}^{-1} \right] M_S Z_{\rm ext} \,. \tag{3.29}$$

Finally, using (3.26) we can rewrite m_ν as

$$m_\nu \equiv \langle\phi^2\rangle \, Z_{\rm ext}^T \left[\frac{1}{\langle\sigma\rangle^2} M_S + \overset{(3)}{Y_\nu} V_N (\delta Z_\kappa D_N^{-1}) V_N^T \overset{(3)}{Y_\nu^T} \right] Z_{\rm ext} \,. \tag{3.30}$$

According to this expression the effects of $d = 5$ operator corrections are reduced to renormalization of masses of the RH neutrinos since δZ_κ is diagonal.

4. Applications of the Dirac structure screening

Here we apply the results obtained in the previous section to several phenomenologically interesting structures of M_S. We study effects of radiative corrections on the light neutrino mass matrix. The matrix M_S will be defined in the basis where the equality of the Yukawa couplings (2.4) is fulfilled.

In the previous sections we have found m_ν in the basis where the neutrino Yukawa coupling matrix Y_ν is diagonal. Now we will discuss m_ν^f — the neutrino mass matrix in the flavor basis where the charge lepton mass matrix Y_e is diagonal. It is related to m_ν as

$$m_\nu^f = U_e^T m_\nu U_e \,, \tag{4.1}$$

where U_e is the transformation of left handed charged lepton components which diagonalizes the matrix Y_e at the electroweak scale. The radiative corrections to Y_e are in general small due to the strong mass hierarchy.

4.1 Quasi-degenerate neutrino spectrum

Let us first consider M_S which is proportional to the unit matrix at Λ, i.e.,

$$M_S = M_S^0 I \,. \tag{4.2}$$

This choice is apparently basis independent and we can take therefore $Y_\nu = Y_N = \mathrm{diag}(y_1, y_2, y_3)$. The right-handed neutrino mass matrix is diagonal and strongly hierarchical:

$$M_N = -Y_N^T M_S^{-1} Y_N \langle\sigma\rangle^2 = \frac{\langle\sigma\rangle^2}{M_S} \mathrm{diag}\left(y_1^2,\, y_2^2,\, y_3^2\right) . \tag{4.3}$$

Therefore $V_N = I$ and we find

$$m_\nu^f \equiv \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} M_S^0 U_e^T Z_{\rm ext}^T \left[I + \delta Z_\kappa\right] Z_{\rm ext} U_e \,. \tag{4.4}$$

The corrections are also diagonal[4]

$$\delta Z_\kappa = \left[\exp\left(\mathcal{A}\,\mathrm{diag}\left(0,\, \ln\frac{y_2^2}{y_3^2},\, \ln\frac{y_1^2}{y_3^2}\right)\right) - I\right] \sim \mathcal{O}(0.1)\,, \tag{4.5}$$

where

$$\mathcal{A} \equiv \frac{1}{16\pi^2}\left(\lambda + \frac{9}{10}g_1^2 + \frac{3}{2}g_2^2\right). \tag{4.6}$$

This leads to splittings of the light neutrino masses which would be degenerate otherwise.

Note that the external corrections (due to the wave function renormalization of the left-handed leptons, eq. (A.1), and the vertex corrections to the neutrino Yukawa couplings, eq. (A.1)), are described in general by off-diagonal matrices due to the mismatch of Y_e and Y_ν structures. As the charged lepton Yukawa couplings are also strongly hierarchical, the largest flavor dependent correction is the one to the 3-3 element. Neglecting the off-diagonal entries, it can be estimated as

$$-2\frac{y_\tau^2}{16\pi^2}\ln\frac{\langle\phi\rangle}{\Lambda} - 4\frac{y_3^2}{16\pi^2}\ln\frac{M_3}{\Lambda} \sim \mathcal{O}(0.1)\,, \tag{4.7}$$

where the second term (due to the neutrino Yukawa coupling) dominates. It has the same order of magnitude as the correction due to the $d = 5$ operator renormalization in eq. (4.5).

Let us now comment on a possibility to explain the neutrino data. In the non-supersymmetric version the mass splitting, Δm, generated by the $d = 5$ operator corrections, $\Delta m = m_0 \delta Z$, leads to $\Delta m^2 = 2m_0\Delta m = 2m_0^2\delta Z = (2-8)\cdot 10^{-3}\,\mathrm{eV}^2$ for the overall scale $m_0 = (0.1 - 0.2)\,\mathrm{eV}$. This can reproduce the atmospheric mass splitting, but it is too large for the solar mass splitting. The ratio of solar and atmospheric mass squared differences,

$$\frac{\Delta m_{21}^2}{\Delta m_{32}^2} \approx \frac{m_2 - m_1}{m_3 - m_2} \sim \mathcal{O}(1)\,, \tag{4.8}$$

does not fit the observations. The external corrections do not improve the situation either. Therefore some other (non-radiative) contribution is required to compensate the 1-2 mass splitting. Mixings can also be generated by small (non-radiative) corrections.

In the supersymmetric version we have $\delta Z_\kappa = 0$ and $Z_{Y_\nu} = I$, so that the mass splitting is produced by the external renormalization only:

$$m_\nu^f = \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} M_S^0 U_e^T Z_{\rm ext}^T U_e Z_{\rm ext}\,. \tag{4.9}$$

In the flavor basis we obtain the mass splitting due to Yukawa couplings coming from the external renormalization:

$$\exp\left[-\frac{1}{8\pi^2}\,\mathrm{diag}(y_e^2,\, y_\mu^2,\, y_\tau^2)\ln\frac{\langle\phi\rangle}{\Lambda}\right], \tag{4.10}$$

[4]We assume a strong hierarchy in Y_ν and use $Y_\nu \sim Y_u$ for the numerical estimates.

where the neutrino Yukawa couplings are neglected. This can provide the atmospheric mass splitting and the mixings should be generated again by correction to the zero order structure.

Next, we consider for M_S the "triangle" structure

$$M_S = M_S^0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{4.11}$$

in the basis where the neutrino Yukawa matrices are diagonal. In the lowest order it produces a degenerate mass spectrum and maximal 2-3 mixing of the light neutrinos. This matrix leads to a spectrum of RH neutrinos with two heavy degenerate states and one relatively light state:

$$M_{1N} = \frac{\langle\sigma\rangle^2 y_1^2}{M_S^0}\,, \qquad M_{2N} = -M_{3N} = \frac{\langle\sigma\rangle^2 y_2 y_3}{M_S^0}\,. \tag{4.12}$$

The renormalization interval "(2)" (see figure 1) is therefore absent and the matrix of $d = 5$ operator corrections can be written as

$$\delta Z_\kappa = \left[\exp\left(\delta \overset{(1)}{Z}_\kappa\right) - 1\right] \mathrm{diag}(0,1,1)\,, \qquad \delta \overset{(1)}{Z}_\kappa = \mathcal{A} \ln \frac{y_1^2}{y_2 y_3}\,, \tag{4.13}$$

where $\mathcal{A}$ is defined in eq. (4.6). The state N_1 decouples and maximal mixing is realized in the 2-3 block of V_N. Using this feature and eq. (4.13) we find from eq. (3.30)

$$m_\nu^f \equiv Z_{\text{ext}}^T \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} M_S^0 U_e^T \overset{(0)}{Z}_\kappa \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 - \delta \overset{(1)}{Z}_\kappa \\ 0 & 1 - \delta \overset{(1)}{Z}_\kappa & 0 \end{pmatrix} Z_{\text{ext}} U_e\,. \tag{4.14}$$

Therefore the $d = 5$ operator corrections do not destroy the triangular structure, but they lead to mass splittings between the degenerate pair and the isolated state:

$$\frac{\Delta m}{m} = \delta \overset{(1)}{Z}_\kappa\ . \tag{4.15}$$

In the supersymmetric version $\delta \overset{(1)}{Z}_\kappa = 0$, so that the original "triangle" structure is renormalized by the external corrections only. In this case, one also needs perturbations of the original screening structure in order to get the correct mixings and mass splittings.

As a third possibility we consider for M_S the "triangle" structure which leads to a degenerate spectrum and maximal 1-2 mixing:

$$M_S = M_S^0 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4.16}$$

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Similar considerations as above results in the mass spectrum of RH neutrinos with two light degenerate states and an isolated heavier state:

$$M_{1N} = -M_{2N} = \frac{\langle\sigma\rangle^2 y_1 y_2}{M_S^0}\,, \qquad M_{3N} = \frac{\langle\sigma\rangle^2 y_3^2}{M_S^0}\,. \tag{4.17}$$

For the light neutrinos we find

$$m_\nu \equiv Z_{\rm ext}^T \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} M_S^0 U_e^T \overset{(0)}{Z}_\kappa \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1-\delta \overset{(2)}{Z}_\kappa \end{pmatrix} Z_{\rm ext} U_e\,, \tag{4.18}$$

where

$$\delta \overset{(2)}{Z}_\kappa = \exp\left(\mathcal{A} \ln \frac{y_1 y_2}{y_3^2}\right) - 1\,. \tag{4.19}$$

The corrections due to running of the $d = 5$ operator are of the same order as in the previous case. The mass splitting

$$\Delta m_{32}^2 = 2m_0 \Delta m_{32} = -2m_0^2 \delta \overset{(2)}{Z}_\kappa = (2-8)\cdot 10^{-3}\,{\rm eV}^2 \tag{4.20}$$

for $m_0 = (0.08 - 0.16)\,$eV can explain the atmospheric neutrino data. The external renormalization contributes in the same way as for $M_S \propto I$.

In the supersymmetric version the "triangle" structure is preserved. Therefore the original matrix M_S as given in eq. (4.16) should be perturbed in order to produce phenomenological acceptable mixings.

Let us comment on the possibility to generate the lepton asymmetry of the Universe via the CP-violating decay of the lightest right handed neutrino(s) [34]. In the case of diagonal M_S the mass of the lightest neutrino N_1 is in the TeV range. So, according to [35] the produced asymmetry is too small to explain the Baryon asymmetry via sphaleron effect. In contrast, in the case of a triangular matrix M_S (4.18) there are two quasi-degenerate RH neutrinos with masses $M \simeq 10^6\,$GeV, and resonant leptogenesis can produce the required asymmetry.

4.2 Perturbations of M_S

Let us consider perturbations of the structure of M_S (which can be required by phenomenology) and effect radiative corrections on these perturbed structures.

As an example we take the matrix

$$M_S = M_S^0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & x & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{4.21}$$

with x being a free parameter. Now the second and third neutrinos are no longer degenerate and the renormalization factor $\overset{(2)}{Z}_\kappa$ in the interval "(2)" between their masses appears.

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Approximating $\overset{(n)}{Z_\kappa}$ by $1 + \mathcal{A}\ln(M_n/M_{n+1})$ we obtain for the light neutrinos

$$m_\nu^f = \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} M_S^0 U_e^T \overset{(0)}{Z_\kappa} Z_{\rm ext}^T \begin{pmatrix} 1 & 0 & 0 \\ \dots & x\,(1+\mathcal{A})\, m_{22}^{\rm th} & 1 + \mathcal{A} m_{23}^{\rm th} \\ \dots & \dots & \mathcal{A} m_{33}^{\rm th} \end{pmatrix} Z_{\rm ext} U_e \,, \tag{4.22}$$

where the threshold dependent corrections, $m_{ij}^{\rm th}$, equal

$$\begin{aligned} m_{22}^{\rm th} =& -3\ln\lambda + \left(\frac{1}{2} - \frac{\lambda^2}{x^2 y}\right) \ln\frac{y-1}{y+1}\,, \\ m_{23}^{\rm th} =& -3\ln\lambda + \frac{1}{2y}\ln\frac{y-1}{y+1}\,, \\ m_{33}^{\rm th} =& \frac{1}{xy}\ln\frac{y-1}{y+1}\,. \end{aligned} \tag{4.23}$$

Here $y \equiv \sqrt{1+4(\lambda/x)^2}$ and $\lambda \equiv y_2/y_3$. (The logarithms depend on the ratios on the RH neutrino masses M_2/M_3.)

The nonzero 3-3 element is generated in eq. (4.22) by the radiative corrections. Furthermore, this element can be enhanced by small parameter x in the denominator, provided that λ is also small enough. Indeed, from eq. (4.23) we find explicitly

$$(m_\nu)_{33} = \begin{cases} \dfrac{2\mathcal{A}}{x}\ln\dfrac{\lambda}{x}\,, & x \gg \lambda \\ -\dfrac{1.26\mathcal{A}}{x}\,, & x = 2\lambda \\ -\dfrac{\mathcal{A}x}{2\lambda^2}\,, & x \ll \lambda \end{cases} \tag{4.24}$$

Since $\mathcal{A} \sim 10^{-2}$, the 3-3 element can be of the order 1 or even bigger if, e.g., $\lambda \ll x < 10^{-2}$. Thus, a quasi-degenerate M_S with nearly maximal 2-3 mixing leads after (non-supersymmetric) renormalization group corrections to the hierarchical mass matrix m_ν with small mixing. The texture (4.21) is not stable against quantum corrections, since the structure of m_ν strongly differs from the original structure of M_S.

This example shows that radiative corrections can substantially modify the original texture of M_S in the light neutrino mass matrix for particular M_S. In other words, radiative corrections may destroy screening.

Apparently the corrections are small if $\lambda \ll x \sim 1$. This corresponds to phenomenologically important case of the dominant 2-3 block:

$$M_S = M_S^0 \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & x & 1 \\ 0 & 1 & x \end{pmatrix} \tag{4.25}$$

with $x \sim 1$ and $\epsilon \ll 1$.

In the supersymmetric version of model screening is stable, since there are no $d = 5$ operator corrections due to non-renormalization theorem. Note also that exact off-diagonal structures of M_S are stable, but small perturbations are unstable with respect to radiative corrections.

4.3 Neutrino symmetry

The mass matrix M_S of the singlets S has no analogy in the quark sector, and in general, it is not related to the quark mass matrices. Furthermore, M_S is generated at a higher scale, than the GUT scale. Therefore it is possible that M_S has certain symmetry which does not show up (or is broken) at lower scales. If screening is realized as discussed in this paper, this symmetry propagates immediately to the light neutrino sector. M_S can therefore be the origin of a specific "neutrino symmetry" which is not seen in the quark and in the charged lepton sectors. So, screening allows us to reconcile special "neutrino" symmetry [9] and the quark-lepton symmetry.

The examples considered in section 4.1 illustrate this possibility, since the matrices (4.2), (4.11), (4.16) have certain symmetries. These symmetries are protected from large corrections in the supersymmetric case, while they can be strongly broken in the non-supersymmetric versions, or when some other interactions and fields beyond the SM or MSSM exist.

4.4 Quark-lepton complementarity

According to the quark-lepton complementarity relation [12 – 15], the lepton mixing is given by maximal mixing minus the quark mixing. This may imply an existence of certain structure in the lepton sector which generates maximal (bi-maximal) mixing and the quark-lepton symmetry in some form. In models with screening mechanism, the mass matrix M_S which has no analogy in the quark sector, can be the origin of the bi-maximal mixing. Then the CKM type mixing follows from the charged lepton mass matrix which is related to the mass matrix of the down quarks, so that $U_e = U_{CKM}$. In the lowest order (without radiative corrections) we find from eq. (4.1)

$$m_\nu^f = \left[\frac{\langle\phi\rangle}{\langle\sigma\rangle}\right]^2 U_e^T M_S U_e = \left[\frac{\langle\phi\rangle}{\langle\sigma\rangle}\right]^2 U_{CKM}^T U_{bm}^* M_S^{\rm diag} U_{bm}^\dagger U_{CKM}\,, \qquad (4.26)$$

where $U_{bm} \equiv R_{23}^m \times R_{12}^m$ is the bi-maximal mixing matrix and R_{ij}^m are the $\pi/4$-rotations in the (ij) planes. So, the leptonic mixing matrix equals $U_{PMNS} = U_{CKM}^\dagger U_{bm}$. This realizes the so called "neutrino scenario" which leads to deviations from the exact quark-lepton complementarity relation [14].

The bi-maximal mixing is produced by the mass matrix [36]

$$M_S^{\rm bimax} = \begin{pmatrix} D-C & B & -B \\ \dots & D & C \\ \dots & \dots & D \end{pmatrix},$$

where B, C, D are arbitrary parameters. As we have found in section 4.2 in the non-supersymmetric case, such a matrix may be unstable with respect to radiative corrections: the $d = 5$ operator corrections can, in particular, suppress the maximal 2-3 mixing. This can be easily seen for vanishing B and small D, when M_S reduces to a triangular structure which is not stable under radiative corrections.

4.5 Singular M_S

Let us consider the special case of singular M_S, $\det M_S = 0$, which can be a consequence of certain symmetry in S sector. Now one can not use immediately (2.3) and the whole double seesaw mass matrix should be considered. In what follows we will show that the tree-level mass matrix of light neutrinos is still proportional to M_S, that is, eq. (2.3) will hold even if M_S is singular. For this we will compare the light neutrino mass spectra in the lowest approximation found from the whole double seesaw matrix $\mathcal{M}$ and from the matrix m_ν after decoupling of the heavy degrees of freedom in eq. (2.3).

According to eq. (2.3) the condition $\det M_S = 0$ implies (at least one) zero eigenvalue in the spectrum of usual LH neutrinos. The same follows from the complete matrix. Indeed,

$$\det \mathcal{M} = -\left(\det Y\right)^2 \det M_S = 0\,,$$

and hence, zero eigenvalue of M_S leads to a massless eigenstate of $\mathcal{M}$. The non-zero eigenvalues of the matrix m_ν , ξ_i, coincide with the eigenvalues of the full matrix $\mathcal{M}$ up to corrections of the order $\langle\phi\rangle / \langle\sigma\rangle$. This can be seen by inserting ξ_i in the characteristic polynomial of the complete matrix $\chi_{\mathcal{M}}[\lambda]$. The result is of order $\mathcal{O}\left(\left(\langle\phi\rangle / \langle\sigma\rangle\right)^8\right) \sim 0$. Hence, ξ_i are to a very good approximation the eigenvalues of $\mathcal{M}$.

There are no other light states, because the expansion of the polynomial

$$\chi_{\mathcal{M}}[\lambda] \prod_i \left(\lambda - \xi_i\right)^{-1}\,,$$

in eigenvalues of the order $\langle\phi\rangle$ does not yield any new solutions. All other eigenvalues are at least of the order $\mathcal{O}(\langle\sigma\rangle^2 / M_S)$.

A peculiarity of the spectrum of $\mathcal{M}$ is the appearance of one heavy Dirac particle, if the eigenstate of M_S with zero mass, S, couples to only one right-handed neutrino N. This Dirac particle is formed by S and N.

The mass spectrum can be easily obtained if $M_S = \mathrm{diag}\left(M_{S1}, M_{S2}, 0\right)$ in the basis where $Y_N = \mathrm{diag}\left(y_1,\, y_2,\, y_3\right)$. Apart from one zero mass which corresponds mainly to ν_3, and two super heavy eigenvalues M_{S1} and M_{S2} for two singlets S, we find

$$m_1 = M_{S1}\frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2}, \quad m_2 = M_{S2}\frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2}, \quad M_1 = -\frac{y_1^2\,\langle\sigma\rangle^2}{M_{S1}}, \quad M_2 = -\frac{y_1^2\,\langle\sigma\rangle^2}{M_{S2}}, \quad M_{DS} = y_3\,\langle\sigma\rangle\,,$$

that is, two light neutrinos predominantly given by $\nu_{1,2}$ with masses m_1 and m_2, two heavy neutrinos mostly consisting of $N_{1,2}$ with masses M_1 and M_2 and one heavy Dirac particle of the GUT scale mass M_{DS} which is formed by N_3 and S_3. The light eigenstates are mainly composed of the left-handed neutrinos and the mixing with other neutral leptons is the order $\mathcal{O}(\langle\phi\rangle / \langle\sigma\rangle)$.

The coincidence of the spectrum of m_ν and the spectrum of light states of $\mathcal{M}$ is related essentially to the fact that the relation between m_ν and M_S is linear, and the characteristic polynomial is linear in the eigenvalues for the non-degenerate case. The same conclusion holds for M_S with two zero eigenvalues.

Let us consider the effect of radiative corrections for this singular case. As long as all contributions to a Majorana mass matrix m_ν receive the same quantum corrections, the RG evolution does not generate non-zero masses from vanishing masses [37]. However, between the mass thresholds of the RH neutrinos, there are two contributions from the decoupling of the RH neutrinos which are renormalized differently. One contribution is due to the $d = 5$ operator decoupled and the other is due to the contribution of the RH neutrinos which are not decoupled yet ($Y_\nu M_N^{-1} Y_\nu^T$) in the intervals $M_2 - M_3$ and $M_1 - M_2$. Hence, the generated mass is proportional to the additional renormalization factor δZ_κ from the $d = 5$ operator between the thresholds and the mismatch between the two mass contributions, i.e. the deviation of the unitary matrix transforming from the eigenbasis of the $d = 5$ operator to the eigenbasis of $Y_\nu M^{-1} Y_\nu^T$ between the thresholds from the unit matrix (See section 4 in [31]). In the supersymmetric version, all contributions to the Majorana mass matrix receive the same quantum corrections, and hence zero mass eigenvalues remain zero.

5. Screening, Grand Unification and flavor symmetry

Let us discuss the possible origin of the screening structure which results from the mass matrix (2.2) together with the condition (2.4). The texture of the matrix (2.2) with zero 1-1, 1-3, and 2-2 blocks can be obtained, e.g., assigning the lepton numbers

$$L(\nu) = L(S) = 1\,, \qquad L(N) = -1\,, \qquad L(\phi) = 0\,, \qquad L(\sigma) = 0$$

so that the lepton numbers of the blocks in the mass matrix (2.2) equal

$$L(\mathcal{M}) = \begin{pmatrix} 2 & 0 & 2 \\ 0 & -2 & 0 \\ 2 & 0 & 2 \end{pmatrix}. \tag{5.1}$$

Then the texture (2.2) appears if the lepton number is only broken by the Majorana mass terms of S. The lepton number can be broken explicitly or spontaneously by the VEV of the new scalar field ρ which has the lepton number $L(\rho) = -2$ and couples with S only: $S^T Y_S S \rho$. (The coupling of Majoron with the SM fields in negligible.) The interaction $\nu^T S \rho$ is forbidden by the gauge symmetry. The possible non-renormalized term

$$\frac{1}{M_{Pl}} L S \phi \rho$$

gives negligible effects due to small VEV $\langle\phi\rangle$. The coupling of ρ with ν is also forbidden by the gauge symmetry. The term $NN\rho$ is absent in the supersymmetric version due to holomorphy.

In the non-supersymmetric version or if also the left superfield ρ^c exists, an extended gauge symmetry can forbid the 2-2 entry. Indeed, in left-right symmetric models N enters the doublet of $\mathrm{SU}(2)_R$ and the 2-2 block has gauge charge (1,3). The corresponding mass term appears once the Higgs triplet Δ_R exists.

The whole texture (2.2) can be a consequence of the gauge symmetry. Let us consider the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)$ symmetry [1, 38]. The $[\mathrm{SU}(2)_L, \mathrm{SU}(2)_R]$ gauge properties of the mass matrix elements are

$$G(\mathcal{M}) = \begin{pmatrix} [3,1] & [2,2] & [2,1] \\ \ldots & [1,3] & [1,2] \\ \ldots & \ldots & [1,1] \end{pmatrix}. \tag{5.2}$$

The required matrix structure is generated if the Higgs bi-doublet with the electroweak VEV, the RH doublet with GU-scale VEV and the singlet with M_{Pl} scale VEV exist. No particular lepton number prescription is needed.

In the context of SO(10) [3], ν and N belong to the 16-plet and S is a singlet. The required texture can be generated by the following Yukawa interactions:

$$Y_\nu\, \mathbf{16} \times \mathbf{16} \times \mathbf{10}_H + Y_N\, \mathbf{16} \times \mathbf{1} \times \overline{\mathbf{16}}_H + Y_S\, \mathbf{1} \times \mathbf{1} \times \mathbf{1}_H\,, \tag{5.3}$$

where $\mathbf{10}_H$, $\overline{\mathbf{16}}_H$, and $\mathbf{1}_H$ are the Higgs multiplets. To generate matrix (2.2) $\mathbf{10}_H$ should acquire the electroweak VEV, $\overline{\mathbf{16}}_H$ - the GU scale VEV in N (SU_5 singlet) direction and $\mathbf{1}_H$ - the Planck scale VEV.

The interactions (5.3) do not produce quark mixing, and the Dirac masses of quarks and leptons are equal at the GUT scale. So, realistic model should contain some additional sources of the fermion masses which may, in general, destroy screening. For instance, one can introduce 126-plet of Higgses. Then VEVs of 126-plet in the "triplet" directions will generate the 1-1 and 2-2 blocks and therefore should be small enough. An alternative is the non-renormalizable interactions of the type $\mathbf{16} \times \mathbf{16} \times \mathbf{16}_H \times \mathbf{16}_H / M_{Pl}$.

Apparently, the interactions (5.3) do not lead to relation (2.4). The equality or proportionality of the Yukawa couplings (2.4) can appear due to

- further unification of ν and S;

- certain flavor symmetry.

Let us comment on these two possibilities.

1. The neutral leptons ν, N and S, can be embedded into a single representation **27** of the gauge symmetry group E_6 [4]. Notice that in this case there are two additional neutral leptons in each generation: S' and S''. The screening structure - the matrix (2.2) with the equality (2.4) can be generated by the couplings

$$Y_{27}\, \mathbf{27} \times \mathbf{27} \times \mathbf{27}_H + Y_{351_S}\, \mathbf{27} \times \mathbf{27} \times (\mathbf{351_S})_H + Y_{351_A}\, \mathbf{27} \times \mathbf{27} \times (\mathbf{351_A})_H\,, \tag{5.4}$$

where the 27-plet as well as the symmetric and antisymmetric 351-plets of Higgses are introduced.

In terms of the maximal subgroup $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{SU}(3)_C \subset E_6$, the leptons transform as $L \sim (\overline{\mathbf{3}}, \mathbf{3}, \mathbf{1})$. The $(\mathrm{SU}(3))^3$ assignment of the neutral leptons is

$$\nu \sim L_3^{\dot{2}}\,, \qquad N \sim L_2^{\dot{3}}\,, \qquad S \sim L_3^{\dot{3}}\,, \qquad S' \sim L_1^{\dot{1}}\,, \qquad S'' \sim L_2^{\dot{2}}\,.$$

The neutral components of Higgs multiplets H, H_A and H_S which can acquire VEVs belong to

$$
\begin{aligned}
H &\subset \big(\overline{\mathbf{3}},\, \mathbf{3},\, \mathbf{1}\big) \subset \mathbf{27}_H \\
H_S &\subset \big(\overline{\mathbf{3}},\, \mathbf{3},\, \mathbf{1}\big) + \big(\mathbf{6},\, \overline{\mathbf{6}},\, \mathbf{1}\big) \subset (\mathbf{351_S})_H \\
H_A &\subset \big(\overline{\mathbf{3}},\, \mathbf{3},\, \mathbf{1}\big) + \big(\overline{\mathbf{3}},\, \overline{\mathbf{6}},\, \mathbf{1}\big) + \big(\mathbf{6},\, \mathbf{3},\, \mathbf{1}\big) \subset (\mathbf{351_A})_H \,.
\end{aligned}
$$

The Higgs multiplets H and H_A generate the Dirac structure[5] and the Majorana mass terms are generated by H_S [23].

Notice that it is not possible to get a Dirac mass term of $S = L_3^{\dot 3}$ with $N = L_2^{\dot 3}$, using $\mathbf{27}_H$ Higgs multiplet due to antisymmetric (in SU(3) indices) interactions. The symmetric Higgs representation $(\mathbf{351_S})_H$ can generate all mass terms of neutral leptons required for screening. Indeed, the VEVs of $H^{\{23\}}_{\{\dot 2\dot 3\}}$ and $H^{\{33\}}_{\{\dot 2\dot 3\}}$ can be of order of the electroweak scale and of the $\mathrm{SU}(2)_R$ breaking scale correspondingly. Also its $H^{\{33\}}_{\{\dot 3\dot 3\}}$ component generates the Majorana mass of S. However, with a single $(\mathbf{351_S})_H$, the structure of matrix M_S will be the same as the structure of the Dirac mass matrices. One can introduce a second $(\mathbf{351_S})_H$ which produces the mass matrix of S of different structure.

Another more promising possibility is to use the antisymmetric $(\mathbf{351_A})_H$ Higgs multiplet which can generate all necessary Dirac matrices. It does not produce the Majorana masses of S which can be done using $(\mathbf{351_S})_H$ so that the structure of M_S is different from that of Dirac structures.

The following VEVs of the $(\mathbf{351_A})_H$ and $(\mathbf{351_S})_H$ components[6]

$$
\begin{aligned}
\left\langle (H_A)_1^{\dot 1} \right\rangle &\simeq \mathcal{O}(\mathrm{SU}(2)_L \text{ breaking scale}) \\
\left\langle (H_A)_1^{\{33\}} \right\rangle &\simeq \mathcal{O}(\mathrm{SU}(2)_R \text{ breaking scale}) \\
\left\langle (H_S)^{\{33\}}_{\{\dot 3\dot 3\}} \right\rangle \simeq \left\langle (H_A)_3^{\dot 3} \right\rangle &\simeq \mathcal{O}(\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \text{ breaking scale})
\end{aligned}
$$

lead to the double seesaw. Indeed, in the basis $(\nu,\, N,\, S,\, S',\, S'')$ they generate mass matrix

$$
\begin{pmatrix}
0 & -Y_{351_A} \left\langle (H_A)_1^{\dot 1} \right\rangle & 0 & 0 & 0 \\
\dots & 0 & -Y_{351_A} \left\langle (H_A)_1^{\{33\}} \right\rangle & 0 & 0 \\
\dots & \dots & Y_{351_S} \left\langle (H_S)^{\{33\}}_{\{\dot 3\dot 3\}} \right\rangle & Y_{351_A} \left\langle (H_A)_1^{\dot 1} \right\rangle & 0 \\
\dots & \dots & \dots & 0 & Y_{351_A} \left\langle (H_A)_3^{\dot 3} \right\rangle \\
\dots & \dots & \dots & \dots & 0
\end{pmatrix}
\tag{5.5}
$$

with the required structure for ν, N and S. In addition to the mass spectrum of the double seesaw, there is one pseudo-Dirac particle formed by S' and S'' with a mass of the order of the $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ breaking scale.

[5]The $\mathbf{27}_H$ and $(\mathbf{351_A})_H$ can not generate Majorana mass terms because the corresponding Yukawa interactions has to be antisymmetric in the SU(3) indices.

[6]The upper indices are $\mathrm{SU}(3)_L$ indices in the fundamental ($\mathbf{3}$) representation and the lower ones belong to $\mathrm{SU}(3)_R$. The $\mathbf{6}$ of SU(3) is represented by symmetric 3×3 matrices and described by 2 symmetrized indices. Dotted indices belong to the complex conjugate representation ($\overline{\mathbf{3}}$). Flavor indices are suppressed.

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Notice that interactions with $\mathbf{27}_H$ Higgs multiplet can be used to generate sub-leading effects, correcting the masses of quarks and producing some deviation from complete screening if needed. Furthermore, as the VEVs of components contributing to the 1-3 and 2-2 element break $SU(2)_L$ invariance, they can only lead to entries of the order of the electroweak scale.

2. The equality of the Yukawa matrices (2.4) can be a consequence of certain flavor symmetry (See reviews [39, 40] and references therein.). It implies that the flavor charges of ν and S are equal and the flavor symmetry uniquely determines the Yukawa couplings of these components with N. For this to happen, the flavor symmetry should be non-abelian.

In the Froggatt-Nielsen (F-N) approach [41] with U(1) flavor symmetry (and in SO(10) context) we can write

$$\left(Z_{ab}\mathbf{16}_a \times \mathbf{16}_b \times \mathbf{10}_H + Z'_{ab}\mathbf{16}_a\mathbf{1}_b \times \overline{\mathbf{16}}_H\right)\left(\frac{H_{FN}}{M_{FN}}\right)^{q_a+q_b}, \tag{5.6}$$

where H_{FN} is the F-N scalar with U(1) charge -1, M_f is the scale where the operators (5.6) are formed and q_a are the U(1) charges of fermionic fields $\mathbf{16}_a$. Z_{ab} and Z'_{ab} are supposed to be couplings of the order 1, but for screening to work one needs to impose an additional proportionality condition $Z_{ab} = cZ'_{ab}$ ($c = const$).

Let us comment on a completely different possibility. If S belongs to the 16-plet and N is the singlet of SO(10), the required equality of the Yukawa couplings (2.4) is automatically reproduced. The screening structure can be generated by interactions

$$Y\mathbf{16} \times \mathbf{1} \times \overline{\mathbf{16}}_H + Y_S\,\mathbf{16} \times \mathbf{16} \times \overline{\mathbf{126}}_H + Y_q\mathbf{16} \times \mathbf{16} \times \mathbf{10}_H\,, \tag{5.7}$$

if $\overline{\mathbf{16}}_H$ has the electroweak VEV in the "ν_L" direction and the GUT scale VEV in the "ν_R" direction, and $\overline{\mathbf{126}}_H$ has the Planck scale VEV in the direction of the SU(5) singlet. The last term in (5.7) gives the Dirac masses of quarks and leptons and also the Dirac mass term for ν and S. The mass matrix generated by (5.7) equals

$$\mathcal{M} = \begin{pmatrix} \sim 0 & Y\left\langle\overline{\mathbf{16}}_H\right\rangle & Y_q\left\langle\mathbf{10}_H\right\rangle \\ Y^T\left\langle\overline{\mathbf{16}}_H\right\rangle^T & \sim 0 & Y^T\left\langle\overline{\mathbf{16}}_H\right\rangle^T \\ Y_q\left\langle\mathbf{10}_H\right\rangle & Y\left\langle\overline{\mathbf{126}}_H\right\rangle & Y_S\left\langle\overline{\mathbf{126}}_H\right\rangle \end{pmatrix}. \tag{5.8}$$

The 126-plet can also contribute to the 1-1 and 1-3 blocks. However, now Y_ν and Y_N are not related to the Dirac matrices of quarks, and the problem of screening does not exist from the beginning. The usual seesaw contribution due to Dirac matrices which are related to the quark mass matrices Y_q is suppressed by large (Planck) scale. That is, here we deal with suppression and not screening of the Dirac structures. Also in this case it would be more natural to identify the RH neutrino with S.

6. Conclusions

1. We studied screening of the Dirac flavor structure in the neutrino masses and mixings. Screening is realized in the context of the double seesaw mechanism if the Dirac type

Yukawa couplings of both seesaw steps are proportional to each other. The flavor structure of the light neutrino mass matrix is determined (in lowest order) by the flavor structure of matrix M_S - the Majorana mass matrix of new singlets at the Planck scale, while the Dirac flavor structure is completely screened. The scale of the neutrino masses is correctly determined by the scales involved in the double seesaw as $M_{Pl} v_{EW}^2 / M_{GU}^2$.

2. The Planck-scale Majorana mass matrix M_S violates lepton number, while at the same time it may have rich flavor symmetries. Screening translates these symmetries of M_S directly into symmetries of m_ν. The structure of M_S can therefore be the origin of specific "neutrino symmetries" which do not show up in the quark sector. In particular, M_S can be degenerate or quasi-degenerate leading (after the inclusion of radiative corrections) to a quasi-degenerate spectrum of light neutrinos. M_S, and consequently m_ν, can also have certain symmetries which result in maximal 2-3 mixing and small 1-3 mixing. A very interesting possibility is that M_S has the bi-maximal structure which explains the quark-lepton complementarity. Screening allows therefore to reconcile the quark-lepton symmetry expressed as (approximate) equality of the Dirac mass matrices of quarks and leptons with the drastically different patterns of quark and lepton mixings. Thus the screening mechanism offers interesting possibilities in GUT model building.

3. The screening mechanism involves the cancellation of mass matrices which arise upon symmetry breaking at very different energy scales, namely the electroweak scale and the GUT scale. These matrices have different gauge properties leading to differences in the radiative corrections which can destroy screening. We studied stability of screening with respect to renormalization group effects.

   It has been found that screening is stable in the MSSM and the external renormalization of the effective $d = 5$ operator can be small. This means that the structure of the light neutrino mass matrix is still mostly determined by M_S, with small radiative corrections after renormalization group running is included.

   In contrast, in the case of the SM, due to the $d = 5$ operator corrections, the screening is unstable for certain structures of M_S. The form of the light neutrino mass matrix m_ν can differ strongly from that of M_S. This can, in fact, be used to explain some features of the light neutrinos via renormalization group effects.

4. Finally, the structure of the mass matrix (2.2) which leads to screening can be obtained using the lepton number or/and gauge symmetry. We outlined how screening could be realized in GUTs. The equality of the Dirac type Yukawa coupling matrices of ν and N and of S and N can be a consequence of certain (horizontal) flavor symmetry or further unification of the fields. The latter can be realized, e.g., within the framework of E_6 gauge models.

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A. Renormalization factors in the extended SM and MSSM

We use GUT charge normalization for the hypercharge, i.e. $\frac{3}{5}\left(g_1^{\mathrm{GUT}}\right)^2 = \left(g_1^{\mathrm{SM}}\right)^2$. The factors Z describing the LL approximation are obtained from the counterterms in ref. [30].

A.1 SM

In the SM extended by RH neutrinos, the wave function renormalization of the RH neutrinos is given by

$$\overset{(n)}{Z}_N = \exp\left(\frac{1}{16\pi^2}\overset{(n)}{Y}{}_\nu^\dagger \overset{(n)}{Y}_\nu \ln\frac{M_n}{M_{n+1}}\right) \tag{A.1}$$

and collecting the contributions from the renormalization of the left-handed doublets

$$\beta^L_{Y_\nu} = \frac{1}{32\pi^2}\left(\overset{(n)}{Y}_\nu \overset{(n)}{Y}{}_\nu^\dagger + Y_e Y_e^\dagger\right),$$

the Higgs doublet

$$\beta^\phi_{Y_\nu} = \frac{1}{32\pi^2}\left(2\,\mathrm{tr}\left(\overset{(n)}{Y}_\nu \overset{(n)}{Y}{}_\nu^\dagger + Y_e Y_e^\dagger + 3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger\right) - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2\right),$$

and the vertex correction to Y_ν

$$\beta^{Y_\nu}_{Y_\nu} = -\frac{1}{8\pi^2}Y_e Y_e^\dagger$$

the external renormalization in the effective theory with n RH neutrinos yields

$$\begin{aligned}\overset{(n)}{Z}_{\mathrm{ext}} = \exp\Bigg(\frac{1}{32\pi^2}\Bigg(&\overset{(n)}{Y}_\nu \overset{(n)}{Y}{}_\nu^\dagger - 3Y_e Y_e^\dagger + 2\,\mathrm{tr}\left(\overset{(n)}{Y}_\nu \overset{(n)}{Y}{}_\nu^\dagger + Y_e Y_e^\dagger + 3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger\right) - \\ &- \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2\Bigg)\ln\frac{M_n}{M_{n+1}}\Bigg).\end{aligned} \tag{A.2}$$

Neglecting the thresholds in the charged lepton sector and the quark sector, the expression for the external renormalization factor $Z^{\mathrm{SM}}_{\mathrm{ext}}$ describing the total external renormalization can be further approximated to

$$\begin{aligned}Z^{\mathrm{SM}}_{\mathrm{ext}} = \exp\Bigg(&\frac{1}{32\pi^2}\sum_{n=0}^{3}\left[\overset{(n)}{Y}_\nu \overset{(n)}{Y}{}_\nu^\dagger + 2\,\mathrm{tr}\left(\overset{(n)}{Y}_\nu \overset{(n)}{Y}{}_\nu^\dagger\right)\right]\ln\frac{M_n}{M_{n+1}} + \\ &+ \frac{1}{32\pi^2}\left[-3Y_e Y_e^\dagger + 2\,\mathrm{tr}\left(Y_e Y_e^\dagger + 3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger\right) - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2\right]\ln\frac{\langle\phi\rangle}{\Lambda}\Bigg).\end{aligned} \tag{A.3}$$

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Here for uniformity of presentation we have denoted

$$M_0 \equiv \langle \phi \rangle \ , \qquad M_4 \equiv \Lambda \, .$$

The renormalization effect due to the additional vertex corrections to the $d = 5$ operator is given by

$$\overset{(n)}{Z_\kappa} = \exp\left(\frac{1}{16\pi^2}\left(\lambda + \frac{9}{10}g_1^2 + \frac{3}{2}g_2^2\right)\ln\frac{M_n}{M_{n+1}}\right) . \tag{A.4}$$

The mass of the right-handed neutrinos receives only corrections from the wave function renormalization to arbitrary loop order.

A.2 MSSM

In the MSSM extended by RH neutrinos, there are no vertex corrections due to the non-renormalization theorem and the wave function renormalization yields

$$\overset{(n)}{Z_L} = \exp\left(\frac{1}{32\pi^2}\left(2Y_e Y_e^\dagger + 2\,\overset{(n)}{Y_\nu}\overset{(n)}{Y_\nu^\dagger} - \frac{3}{5}g_1^2 - 3g_2^2\right)\ln\frac{M_n}{M_{n+1}}\right) \tag{A.5}$$

$$\overset{(n)}{Z_N} = \exp\left(\frac{1}{8\pi^2}\,\overset{(n)}{Y_\nu^\dagger}\overset{(n)}{Y_\nu}\ln\frac{M_n}{M_{n+1}}\right) \tag{A.6}$$

$$\overset{(n)}{Z_\phi} = \exp\left(\frac{1}{32\pi^2}\left(\mathrm{tr}\left(6Y_u Y_u^\dagger + 2\,\overset{(n)}{Y_\nu}\overset{(n)}{Y_\nu^\dagger}\right) - \frac{3}{5}g_1^2 - 3g_2^2\right)\ln\frac{M_n}{M_{n+1}}\right) . \tag{A.7}$$

The external renormalization factor $Z_{\mathrm{ext}}^{\mathrm{MSSM}}$ is given by the product of the wave function renormalization of the left-handed doublet with the Higgs doublet

$$\overset{(n)}{Z_{\mathrm{ext}}} = \overset{(n)}{Z_L}\overset{(n)}{Z_\phi} \tag{A.8}$$

because the two wave function renormalization factors commute. As the neutrino Yukawa couplings only change at the thresholds (up to 1 loop order), the external renormalization factor can be further approximated by

$$\begin{aligned} Z_{\mathrm{ext}}^{\mathrm{MSSM}} &= \exp\left(\frac{1}{16\pi^2}\sum_{n=0}^{3}\left(\overset{(n)}{Y_\nu}\overset{(n)}{Y_\nu^\dagger} + \mathrm{tr}\left(\overset{(n)}{Y_\nu}\overset{(n)}{Y_\nu^\dagger}\right)\right)\ln\frac{M_n}{M_{n+1}} + \right. \\ &\qquad \left. + \frac{1}{16\pi^2}\left(Y_e Y_e^\dagger - \frac{3}{5}g_1^2 - 3g_2^2 + 3\,\mathrm{tr}\left(Y_u Y_u^\dagger\right)\right)\ln\frac{\langle\phi\rangle}{\Lambda}\right) . \end{aligned} \tag{A.9}$$

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